sciences, and history, play a much greater part than in antiquity. Result: mysticism and agnosticism. Prof. Dorner combats both. The physical sciences themselves point the way to metaphysical principles; the problem of philosophy is not merely epistemology or the making of a world-conception out of the disparate elements of knowledge and experience, but is rather the search for a unified metaphysic by which the fundamentals of the world and of the spiritual life may equally be grounded in an Absolute Being.

AMERICAN TEXT-BOOKS OF MATHEMATICS.

(1) *College Algebra.* By Prof. H. L. Reitz and A. R. Crathorne. Pp. xiii+261. (New York: H. Holt and Co.; London: G. Bell and Sons, 1909.) Price 6*s.*

(2) *Trigonometry.* By Prof. A. G. Hall and F. G. Frink. Pp. x+146+93. (New York: H. Holt and Co.; London: G. Bell and Sons, 1909.) Price 7*s.* 6*d.*

(3) *First Course in Calculus.* By Prof. E. J. Townsend and Prof. G. A. Goodenough. Pp. xii+466. (New York: H. Holt and Co.; London: G. Bell and Sons, 1908.) Price 12*s.*

THESE books are the first three of a series which is intended in the first place for students taking a university course in engineering, and also, to a certain extent, for mathematical students. It will be noticed that each book has two authors, who have been selected to represent the interests of readers of both classes.

(1) and (2). The chief novelty in these books is to be found in the variety of examples, selected from very different subjects. Thus, as an example on evaluating algebraic expressions ("Algebra," p. 24), the student is asked to verify in a few cases a formula for the day of the week, which (after an obvious simplification) can be written [1]—

$$2+p+2q+[\tfrac{3}{5}(q+1)]+s+[\tfrac{1}{4}s]-2r+[\tfrac{1}{4}r]\equiv t \pmod{7}$$

where t is the day of the week (Sunday being 1 and Saturdav 7), and the date is the pth day of the qth month in the year $100r+s$. The reader interested in such matters may find it instructive to reconstruct this formula, of which the most interesting feature is the part depending on q; it will be found that starting from March (and ignoring February) the lengths of the months recur after intervals of five months, and this is the basis of the formula.

The problems proposed in the trigonometry are chosen so as to illustrate the practical difficulties of surveying so far as possible. Great stress is laid on the advantage of making a *form* for numerical calculations, before starting to use the tables at all. One useful consequence is that, in the typical examples worked out, the logarithms to be added are placed in *vertical* columns, as would be done in practical work; writers of text-books are very apt (in order to save space) to arrange such logarithms *horizontally*. The result is that imitative readers are liable to arrange their work in the same way, with disastrous results.

The last ninety-three pages in the trigonometry contain a good set of five-figure tables. The table of logarithmic functions, however, makes no special provision for finding the log sin and log tan of *small* angles; a very simple rule applies to four-figure or five-figure tables (with a difference of 1′ in angle), namely—

$$\log \sin \theta = \log \sin \alpha + (\log \theta - \log \alpha)$$

and this (or some similar rule) ought to be given in all tables which do not provide a special table for the first few degrees. The table of squares is interesting, as it gives the *exact* squares from 1 to 1000^2, without occupying more space than an ordinary four-figure table; this is effected by following the arrangement of Crelle's multiplication tables, where every number in the same horizontal line is terminated by the same two digits. Both in the algebra and trigonometry certain of the best-known power-series are given and used for numerical calculations; but the authors of the algebra are content to refer to the calculus (No. 3) for proofs, while in the trigonometry some proofs are provided, which would not be accepted nowadays. It might be better definitely to cut out all such proofs from books on trigonometry; in modern teaching the elements of the calculus are couainly regarded as easier (and more generally useful) than the "calculus-dodging" of the old-fashioned books.

(3) Compared with recent English books having similar titles this book contains fuller treatment of the applications of the calculus to applied mathematics; for instance, centroids, moments of inertia, resultant fluid pressure, are considered at some length, as exercises on integration.

As in the other books of the series, a large variety of illustrative examples will be found; thus the exponential function is connected with the chemical problem of inversion of cane-sugar. The theory of maxima is illustrated by the efficiency of a rough screw, the speed of signalling in a cable, and the h.p. transmitted by a hemp-rope.

In dealing with the Taylor's series derived from a given function, care is taken to point out that the series *may* converge without being equal to the function; this is a point quite commonly overlooked in the theory, and possibly an example would have helped to emphasise it.[1]

As might be anticipated from the character of the series, a good deal of stress is laid on methods for approximate integration, such as Simpson's rules and other similar methods, and several examples are given of their application to irregular solids such as rails. It seems strange, however, that the *exact* form of Simpson's rule is not mentioned, for finding the volume and centroid of a railway embankment (or the slice of an ellipsoid) in terms of the areas of the ends and the area of the central section.

The use of infinite series for finding an integral

[1] The notation is that of the theory of numbers: that is, $[x]$ denotes the integral part of x, and $y \equiv z$ means that $y-z$ is divisible by 7. Note that January and February are regarded as belonging to the *previous* year, with the values $q=13, 14$.

[1] Thus, Pringsheim's function $\Sigma \frac{(-1)^n a^n}{n!(1+x^2a^{2n})}$ has the Taylor's series $e^{-a} - x^2e^{-a^3} + x^4e^{a^5} \ldots$: but if $a>1$, although both series converge for all *real* values of x, they are unequal except for $x=0$. For instance, if $a=2$, $x=2$, it will be found that the first series is less than 0·109, while the second is greater than 0·133; on the other hand if $a=\frac{1}{2}$, $x=\frac{1}{2}$, both series are equal to 0·434 (nearly).

is also classed by the authors as "approximate integration"; this is a view which does not seem altogether satisfactory. At any rate, the nature of the approximation involved in using an infinite series is certainly different from that associated with the use of Simpson's rules. Incidentally, at least one example (p. 379), in which the integration is effected by a series $\left(\int\{y/(y+c)\}dS\right.$ integrated over a circle), is easily reduced to finite terms in the form,

$$\pi a^2 - 2\pi c\,\{c-(c^2-a^2)^{\frac{1}{2}}\}.$$

Some of the integrals proposed for evaluation by the aid of series are not very easy to evaluate *directly*; for instance (p. 380), the elliptic integrals,

$$\int_0^x \frac{dx}{\sqrt{(\sin x)}} \text{ and } \int_0^1 \frac{dx}{\sqrt{(1-\frac{1}{4}x^4)}}.$$

Both of these can be expressed in various forms, but the series which are more immediately suggested are not very suitable for ordinary calculations; in particular the second of them suggests the binomial expansion of $(1-x^4)^{-\frac{1}{2}}$, but the resulting series is quite hopeless for numerical work. Of course, there are several ways of transforming the integrals before conversion to series; but such transformations might well be suggested in the questions, or the reader may not succeed in guessing what to do first.

In reading the chapters on applications to plane curves one cannot help regretting some of the old-fashioned geometrical types of proof; no doubt the older books contain much that is not only unsound, but incapable of being made sound. But in spite of this, a geometrical treatment is more attractive to the ordinary reader, and in many cases the proofs can be made reasonably accurate by the aid of very little additional analysis.

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BOOKS ON NATURE-STUDY.

(1) *Der Naturfreund am Strande der Adria und des Mittelmeergebietes.* By Prof. Carl I. Cori. Pp. viii+148+22 plates. (Leipzig: Verlag von Dr. Werner Klinkhardt, 1910.) Price 3.50 marks.

(2) *The Aims and Methods of Nature-Study. A Guide for Teachers.* By Dr. John Rennie. With an introduction by Prof. J. Arthur Thomson. Pp. xvi+352. (London: W. B. Clive, University Tutorial Press, Ltd., 1910.) Price 3*s.* 6*d.*

(1) PROF. CORI'S volume is not intended to give descriptions of the systematic characters and organisation of marine creatures, but rather to be a companion to direct the attention of the nature-student to the more commonly occurring marine organisms and to the chief phenomena associated with them. After a brief account of the past history of the Mediterranean and Adriatic, the author passes to the consideration of the animals of the beach—Arenicola, Sipunculus, Solen, Venus, Echinocardium, Synapta, Carcinus, &c.—the chief features and theoretical points of interest associated with many of which are indicated. While dealing with Annelids, the author directs attention to their relationship to the Crustacea and to the theory of the Annelid ancestry of vertebrates. Modifications of structure correlated with certain habitats, as illustrated, for example, by sessile molluscs, and the habits of animals, *e.g.* the shamming death and autotomy of crabs, are dealt with in an interesting manner. The description of the abundance of life on the beach leads up to remarks on the origin of life in shallow water, "die Geburtsstätte alles Seins." The lagoons and their flora and fauna—Mysidæ, Carcinus, Cardium, Labrax, Anguilla, &c.—and the Zostera meadows, with their extensive and characteristic fauna—Virbius, Spadella, Turbellaria, Cerianthus, Sepia, pipe-fish, sea-horses, &c.—are the subjects of two chapters.

The account of Sepia contains interesting references to the antiquity and former greater abundance of species of Cephalopods in the period when the Ammonites flourished, and to the power of colour change, owing to which "spiegelt sich sozusagen die Seele der Sepia auf ihrer Haut ab." Throughout the volume the author brings before the reader observations on the mode of life, the food and special points in the physiology of the animals under description; for instance, he points out that in Trachinus, the weever-fish, the spreading of the spines and the assumption of the defensive attitude are dependent chiefly on stimulation of the tail. The organisms of the rocks and rock-pools are then considered, attention being given to boring animals, *e.g.* Pholas, Paracentrotus, the former boring by chemical, the latter by mechanical means.

The concluding chapters give accounts of the larger organisms obtained by dredging, and in the plankton (Rhizostoma, some Siphonophores, Ctenophores, and Salps) and on the high sea (fishes, dolphins, &c.). The figures are for the most part excellent, but a few, for instance, those of Aphrodite, Arenicola, and Balanoglossus on Taf. vi., are capable of improvement. A few errors of spelling occur in the names of the animals figured, *e.g.* *pilleata* (for *pileata*), *forscalea, Litorina,* and *Echineis.* But these are only slight blemishes, and do not seriously detract from the value of this excellent work, which cannot fail to stimulate the interest and imagination of the nature-lovers for whom it is intended.

(2) Dr. Rennie aims at imparting a definite continuity of principle to the teaching of the subject of nature-study and to this end he outlines series of carefully graded courses. He holds rightly that the facts need to be carefully grouped or arranged in sequence, according to principle, in the mind of the teacher (although the principle need not always be enunciated to the pupils), for only in this way can the teaching be effective. Four school courses are suggested, namely, for pupils of seven or eight years, eight or nine years, nine to twelve years, and seniors, all of which are arranged on a seasonal plan and deal in turn with plants, animals, weather studies, calendars, and general considerations. Several chapters are devoted to excellent object-lessons on common living things, *e.g.* frogs and toads, birds and their eggs and feathers, the mole, shells, the snail, caterpillars and moths, earthworms, gnats, buttercups, common fruits and seeds, trees, ferns, &c. Then follow elementary studies of some common rocks, suggestions for a